this is a greedy algorithm for a discrete optimization problem, there is no guarantee that the global maximum will be reached. However, the theorem demonstrates that the effective independence distribution is an exact measure of the information that will be lost when deleting one potential sensor location, and in practical applications the method has been shown[1,2] to be effective for optimally locating sensors.

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Reply by Author to W. L. Poston and R. H. Tolson

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THE author would like to thank Poston and Tolson for their interest in the effective independence method of sensor placement he presented in Ref. 1. It was believed that the effective independence method ranked candidate sensor locations such that the deletion of the lowest ranked sensor resulted in the smallest change in the determinant of the Fisher information matrix (FIM). However, this hypothesis had not been proven prior to Poston and Tolson's comment. Their result is elegant and its proof convincing; however, the author would like to present an alternate proof recently brought to his attention by L. Yao, a graduate student in the Department of Electrical and Computer Engineering at the University of Wisconsin. The proof starts with a lemma from Ref. 2.

Lemma: Let $C \in R^{n\times m}$, $D \in R^{m\times n}$, and I_p be a $p \times p$ identity matrix. Then

$$\det(I_n - CD) = \det(I_m - DC) \tag{1}$$

Proof: See the Appendix in Ref. 2. □

The following theorem states Poston and Tolson's result in a slightly different way:

Theorem: $\forall A = \Phi^T\Phi \in R^{n\times n}$ and A positive definite. Let $r_i \in R^{1\times n}$ be the ith row vector of Φ and $B = A - r_i^T r_i$. Then

$$\det(B) = \det(A)(1 - E_{Di}) \tag{2}$$

where $0 \le E_{Di} \le 1$.

Note that $r_i = R_i^T$, which is used in both the comment and Ref. 1.

Proof:

$$\begin{aligned}\det(B) &= \det(A - r_i^T r_i)\\ &= \det\left[A(I - A^{-1} r_i^T r_i)\right]\end{aligned} \tag{3}$$

Since A and $(I - A^{-1} r_i^T r_i)$ are both square matrices,

$$\begin{aligned}\det(B) &= \det(A)\det(I - A^{-1} r_i^T r_i)\\ &= \det(A)\det(1 - r_i A^{-1} r_i^T) = \det(A)(1 - E_{Di})\end{aligned} \tag{4}$$

where the foregoing lemma has been used and $E_{Di} = r_i A^{-1} r_i^T$. Recall that

$$B = A - r_i^T r_i = \sum_{\substack{j=1\\ j\ne i}}^{n} r_j^T r_j = \Gamma_i^T \Gamma_i \tag{5}$$

where $\Gamma_i = [r_1^T, \ldots, r_{i-1}^T, r_{i+1}^T, \ldots, r_n^T]^T$. Since matrix B can be expressed in this factored form, it must be positive semidefinite. This implies that $\det(B) \ge 0$ and thus $E_{Di} \le 1$. Because A is assumed to be positive definite, A^{-1} is also positive definite. Therefore, $\forall r_i \ne 0$, $i = 1, \ldots, n$,

$$E_{Di} = r_i A^{-1} r_i^T > 0 \tag{6}$$

However, r_i could be a zero row in Φ; therefore, $E_{Di} \ge 0$. This completes the proof. □

Thus, the effective independence sensor placement method iteratively deletes candidate sensor locations that have the smallest impact on the value of the Fisher information matrix determinant.

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Errata

Minimizing Selective Availability Error on Satellite and Ground Global Positioning System Measurements

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BEGINNING with the first full paragraph on page 1308 and continuing on page 1309, six paragraphs appear out of order. The AIAA Editorial Staff regrets this error and any inconvenience it has caused our readers. The correct order appears below:

Since carrier-phase data noise is intrinsically low, smoothing over the entire 5-min integration time is not necessary. Instead of removing the satellite dynamics with a good model, a low-order polynomial interpolation over a short time period can be used for the compression of carrier phase. The simulation analysis in the following section indicates that for Topex data at 1-s intervals, a cubic interpolation over four points every 5 min is appropriate even with the strong Topex dynamics. For ground data, a compression scheme with a quadratic interpolation over three 10-s data points every 5 min can be adopted. This is based on the fact that the dynamics are much lower and the corrected time-tag for the ground data will always be less than 0.1 s away from the nearest raw data point, hence a quadratic fit would be appropriate; the corrected time-tag for Topex could be as large as 0.5 s away from the nearest raw data point and a cubic interpolation is recommended to assure a low interpolation error. The following simulation analysis will demonstrate that the differential SA effects between Topex and ground data are small despite different interpolation polynomials used.

The higher data noise in pseudorange discourages the use of any data decimation. However, since the precise carrier phase has identical satellite dynamics as the pseudorange, it can be treated as a dynamics model and subtracted from the pseudorange. Then the dynamics-removed pseudorange can be compressed with a simple averaging over the entire 5-min period. The dynamics is later recovered by adding the compressed carrier phase to the compressed pseudorange. This scheme is called smoothing of pseudorange using carrier phase[5] and has been used in data-acquisition software in some GPS receivers. Since the removal of dynamics using carrier phase also removes the SA effects, the compression through averaging of pseudorange does not introduce any additional interpolation error. Hence, the compressed differential pseudorange will have the same residual SA effects as the compressed differential carrier phase, which are low due to the small (1-s) data intervals. Note that it is important to maintain identical SA effects in pseudorange and carrier-phase data types since they are later removed as common clock error in the filtering process (which is comparable to differencing). A flow diagram summarizing the data compression scheme is shown in Fig. 1.

Simulation Analysis

To assess the effectiveness of the proposed data compression scheme in reducing the SA effects, a simulation analysis was carried out. The orbits of Topex and a constellation of 18 GPS satellites in six orbital planes were computed over a 2-h period. Simulated "raw data," at 1-s intervals for Topex carrier phase and 10-s intervals for Topex pseudorange and both data types for six globally distributed ground receivers, were generated. The SA effects for the 18 GPS satellites were simulated and added to the simulated data. All ground receivers' clocks were assumed to be perfect, whereas different levels of Topex clock offsets were considered.

The gross differential SA effects without correcting for the time-tag offset were first assessed. The raw data without including satellite dynamics were generated and then decimated to 5-min data points. Differenced data were formed between Topex and ground receivers observing the same GPS satellites. The rms effects of 18 passes of differenced data observing 10 different GPS satellites having the longest common view periods, over the entire 2-h orbital cycle, were computed and plotted in Fig. 2 for three different Topex clock offsets: 0.5, 2.5, and 4.5 s. The residual effects on carrier phase for the three Topex clock offsets are the same as those on pseudorange at 0.5-s offset since Topex carrier phase is sampled at 1-s intervals. Without proper correction for the time-tag offsets, the residual SA effects on the differenced data are indeed large: 6 cm for carrier phase and about 12 cm for each second of Topex clock offset for pseudorange, up to 60 cm at the maximum offset of 5 s. Note that these are the rms values over the 18 passes; at times the errors are a factor of 3 higher. The residual SA effects without time-tag correction for a typical pass of differential pseudorange between Topex and a ground receiver are shown in Fig. 3.

Next, we investigated the effectiveness of the data compression scheme for reducing the SA effects using different interpolation strategies (linear, quadratic, and cubic). Satellite dynamics were now included in the raw data. The data were compressed and differenced between Topex and ground receivers. These results are then differenced from the "truth" models where the SA effects are turned off. The rms residual errors are summarized in Table 1. A linear interpolation results in a large error (75 cm) due to poor modeling of the satellite dynamics, and thus should not be adopted. With the proposed data compression scheme the effects on both pseudorange and carrier phase are reduced to 0.4 mm using a quadratic interpolation and to <0.1 mm using a cubic interpolation. Although the difference is negligible for pseudorange, cubic interpolation clearly is superior over quadratic for carrier phase, which has a low data noise comparable to the 0.4-mm level. Hence, a cubic interpolation of carrier phase is recommended for Topex data compression. A similar comparison was also made (Table 2) for ground data assuming a 20-ms clock offset due to light-time difference. Because of lower dynamics and small time offset, the error is only 7 mm with a linear interpolation. It reduces to below 0.1 mm with either a quadratic or a cubic interpolation. Hence, a quadratic interpolation of carrier phase is appropriate for ground data compression.

Conclusions

The proposed data compression scheme reduces the selective availability error on satellite and ground differential GPS measurements from 1 m to below 0.1 mm. The residual error will be increased when selective availability of a higher level is turned on. However, the residual error will still be lower than 1 mm even with an increase in the level by an order of magnitude. Although a specific satellite, Topex/Poseidon, has been used as an illustrating example, the scheme is readily applicable to other satellites with similar data-acquisition scenario. The data compression scheme involves only simple algorithms which are computationally efficient. Hence, it can conveniently be incorporated into the data-acquisition software of GPS receivers, where the "raw" data sampling rates are dictated. In general, the proposed scheme applies to non-real-time satellite orbit determination. Real-time applications would require accurate GPS orbits independent of the broadcast ephemerides, and data transmission between receivers for real-time data differencing.